MATH-2212: Calculus of One Variable II Worksheet for sections 10.1, 10.2

Worksheet for Sections 10.1, 10.2 – Parametric Curves

# Some Useful Formulas

## Tangent Lines to Parametric Curves

An equation of the tangent line to the graph of a parametrically defined curve *x* = *x*(*t*), *y* = *y*(*t*) at a point *t*0, i.e. at a point (*x*0 = *x*(*t*0)*, y*0 = *y*(*t*0)), can be set up as

d*y*

*y − y* =

0

d*x*I

d*y*

*·* (*x − x* ) , where

I

0

d*x*

d*y/* d*t*

=

d*x/* d*t*

d*x*

, provided */*= 0*.*

d*t*

*t*=*t*0

Alternatively, an equation of such tangent line can be set up as

*yi*(*t*0) *·* (*x − x*0) *− xi*(*t*0) *·* (*y − y*0) = 0*,* provided *xi*(*t*0) and *yi*(*t*0) are not both zero.

## Areas Under Parametric Curves

If a curve is given by parametric equations *x* = *x*(*t*)*, y* = *y*(*t*) for *α* � *t* � *β* that trace out the curve once from left to right, then the area enclosed between this curve and the *x*-axis is

Area =

*β*

*y*(*t*)*xi*(*t*) d*t.*

*α*

## The Arc Length of a Parametric Curve

For a parametric curve *x* = *x*(*t*), *y* = *y*(*t*), where *a* � *t* � *b*, the arc length is:

*b* 2 2

*L* = [*xi*(*t*)] + [*yi*(*t*)]

*a*

d*t.*

# Practice problems

1. Express the given parametric curve by an equation in *x* and *y*, and then sketch the curve.
   1. *x* = 8*t*2 *−* 5, *y* = 2*t −* 1, for *t ∈* (*−∞,* +*∞*)
   2. *x* = 2 sin *t* + 1, *y* = 3 cos *t −* 2, for 0 � *t* � 2*π*
   3. *x* = 2 sin *t* + 1, *y* = 4 sin *t* + 3, for 0 � *t* � 2*π*

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1. Find an equation of the tangent line to the given curve at the given point.
   1. *x* = 2*e*2*t* + 1, *y* = 3*e*3*t* + 2; at *t* = 0
   2. *x* = *√t* 3, *y* = 2 *t*; at *t* = 7

*− −*

*t −* 6

*t*4

* 1. *x* = *t*2 *−* 3, *y* =

+ *t* + 1; at (1*,* 3)

4

1. Find the area enclosed by the *x*-axis and the curve *x* = *t*3 + 1, *y* = 2*t t*2. *(Hint: First, find the intercepts of this curve with the x-axis by solving the equation y* = 0*.)*

*−*

1. Find the length of the given parametric curve.
   1. *x* = 3*t*2 *−* 4, *y* = 2*t*3 + 5, for *t ∈* [0*,* 2]

*π π*

* 1. *x* = cos *t* + sin *t*, *y* = cos *t −* sin *t*, for *−* 2 � *t* � 2

*π*

* 1. *x* = cos *t* + *t* sin *t*, *y* = sin *t − t* cos *t*, for 0 � *t* � 2